
<u>QUESTION 1.</u> (30 points for 15 short questions, 2 points for each short question)

Show, in a few steps in the writing sheets, your method of solution. Write your final answers in the answer sheets provided. Partial credits will be given for answers without showing method of solution.

1.1 For an observer at latitude 42.5^o N and longitude 71^o W, estimate the time of sun rise on 21 December if the observer's civil time is -5 hours from GMT. Ignore refraction of the atmosphere and the size of the solar disc.

1.2 The largest angular separation between Venus and the Sun, when viewed from the Earth, is 46^o. Calculate the radius of Venus's circular orbit in A.U.

1.3 The time interval between noon on 1 July and noon on 31 December is 183 solar days. What is this interval in sidereal days?

1.4 One night during a full Moon, the Moon subtends an angle of 0.46 degree to an observer. What is the observer's distance to the Moon on that night?

1.5 An observer was able to measure the difference in the directions, due to the Earth's motion around the Sun, to a star as distant as 100 parsecs away. What was the minimum angular difference in arc seconds this observer could measure?

1.6 A Sun-orbiting periodic comet is the farthest at 31.5 A.U. and the closest at 0.5 A.U.. What is the orbital period of this comet?

1.7 For the comet in question 1.6, what is the area (in square A.U. per year) swept by the line joining the comet and the Sun?

1.8 At what wavelength does a star with the surface temperature of 4000 K emit most intensely?

1.9 Calculate the total luminosity of a star whose surface temperature is 7500 K, and whose radius is 2.5 times that of our Sun. Give your answer in units of the solar luminosity, assuming the surface temperature of the Sun to be 5800 K.

1.10 A K star on the Main Sequence has a luminosity of $0.4\,L_{\odot}$. This star is observed to have a flux of 6.23×10^{-14} $\mathrm{W.m^{-2}}$. What is the distance to this star? You may neglect the atmospheric effect.

1.11 A supernova shines with a luminosity 10^{10} times that of the Sun. If such a supernova appears in our sky as bright as the Sun, how far away from us must it be located?

1.12 The (spin-flip) transition of atomic hydrogen at rest generates the electromagnetic wave of the frequency $\nu_0 =$ 1420.406 MHz. Such an emission from a gas cloud near the galactic center is observed to have a frequency $\nu = 1421.65$ MHz. Calculate the velocity of the gas cloud. Is it moving towards or away from the Earth?

1.13 A crater on the surface of the Moon has a diameter of 80 km. Is it possible to resolve this crater with naked eyes, assuming the eye pupil aperture is 5 mm ?

1.14 If the Sun were to collapse gravitationally to form a non-rotating black hole, what would be its event horizon (its Schwarzschild radius)?

1.15 The magnitude of the faintest star you can see with naked eyes is $m = 6$, whereas that of the brightest star in the sky is $m = -1.5$. What is the energy-flux ratio of the faintest to that of the brightest?

QUESTION 2 A PLANET & ITS SURFACE TEMPERATURE (10 points)

A fast rotating planet of radius R with surface albedo α is orbiting a star of luminosity L. The orbital radius is D. It is assumed here that, at equilibrium, all of the energy absorbed by the planet is re-emitted as a blackbody radiation.

a.) What is the radiation flux from the star at the planet's surface? (1.5 points)

b.) What is the total rate of energy absorbed by the planet? (1.5 points)

c.) What is the reflected luminosity of the planet? (2 points)

d.) What is the average blackbody temperature of the planet's surface? (2 points)

e.) If we were to assume that one side of the planet is always facing the star, what would be the average surface temperature of that side? (2 points)

f.) For the planet in problem d:

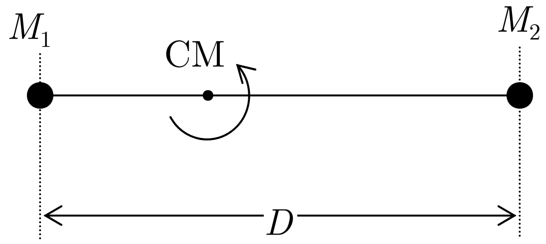
$\alpha = 0.25$,

$D = 1.523$ A.U.,

calculate its surface temperature in kelvins for the value of

$L = 3.826 \times 10^{26}$ W. (1 point)

QUESTION 3 BINARY SYSTEM (10 points)



A binary star system consists of M_1 and M_2 separated by a distance D. M_1 and M_2 are revolving with an angular velocity ω in circular orbits about their common centre of mass. Mass is continuously being transferred from one star to the other. This transfer of mass causes their orbital period and their separation to change slowly with time.

In order to simplify the analysis, we will assume that the stars are like point particles and that the effects of the rotation about their own axes are negligible.

a) What is the total angular momentum and kinetic energy of the system? (2 points)

b) Find the relation between the angular velocity ω and the distance D between the stars. (2 points)

c) In a time duration Δt, a mass transfer between the two stars results in a change of mass ΔM_1 in star M_1, find the quantity $\Delta\omega$ in terms of ω, M_1, M_2 and ΔM_1. (3 points)

d) In a certain binary system, $M_1 = 2.9\ \mathrm{M}_\odot$, $M_2 = 1.4\ \mathrm{M}_\odot$ and the orbital period, $T = 2.49$ days. After 100 years, the period T has increased by 20 s. Find the value of $\dfrac{\Delta M_1}{M_1 \Delta t}$ (in the unit "per year"). (1.5 points)

e) In which direction is mass flowing, from M_1 to M_2, or M_2 to M_1? (0.5 point)

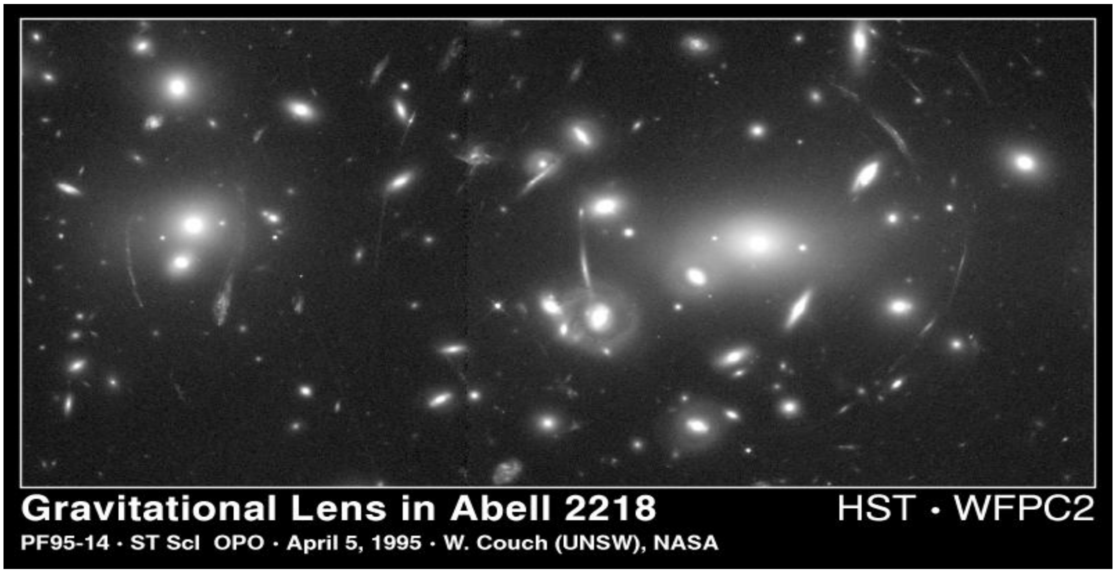
f) Find also the value of $\frac{\Delta D}{D\Delta t}$ (in the unit "per year"). (1 point)

You may use these approximations:

$(1+x)^n \sim 1+nx$, when $x \ll 1$;

$(1+x)\left(1+y\right) \sim 1+x+y$, when $x \ll 1$, $y \ll 1$.

QUESTION 4 **GRAVITATIONAL LENSING (10 points)**



The deflection of light by a gravitational field was first predicted by Einstein in 1912 a few years before the publication of the General Relativity in 1916.

A massive object that causes a light deflection behaves like a classical lens.

This prediction was confirmed by Sir Arthur Stanley Eddington in 1919.

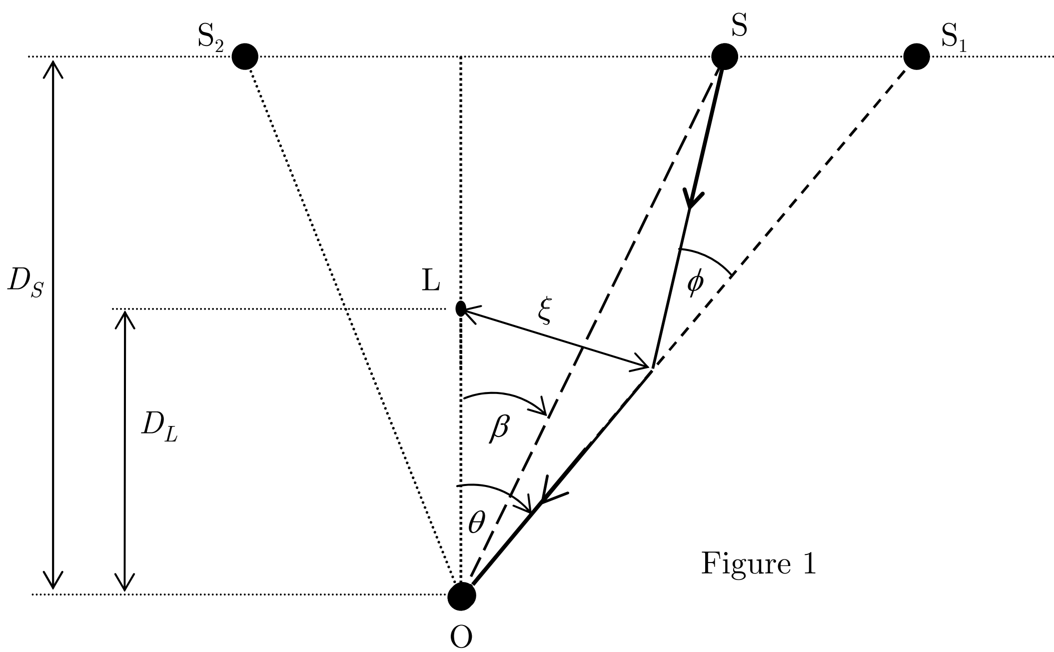


Figure 1

Consider a spherically symmetric lens, with a mass M with an impact parameter ξ from the centre. The deflection equation in this case is given by:

$$\phi = \frac{4GM}{\xi c^2} \quad \text{, a very small angle}$$

In figure 1, the massive object which behaves like a lens is at L. Light rays emitted from the source S being deflected by the lens are observed by observer O as images S_1 and S_2. Here, ϕ, β, and θ are very very small angles.

a) For a special case in which the source is perfectly aligned with the lens such that $\beta = 0$, show that a ring-like image will occur with the angular radius, called Einstein radius θ_E , given by:

$$\theta_E = \sqrt{\left(\frac{4GM}{c^2}\right)\left(\frac{D_S - D_L}{D_L D_S}\right)} \qquad \text{(2 points)}$$

b) The distance (from Earth) to a source star is about 50 kpc. A solar-mass lens is about 10 kpc from the star. Calculate the angular radius of the Einstein ring formed by this solar-mass lens with the perfect alignment. (1 point)

c). What is the resolution of the Hubble space telescope with 2.4 m diameter mirror? Could the Hubble telescope resolve the Einstein ring in b)? (2 points)

d). In figure 1, for an isolated point source S, there will be two images $\left(S_1 \text{ and } S_2\right)$ formed by the gravitational lens. Find the positions (θ_1 and θ_2) of the two images. Answer in terms of β and θ_E. (2 points)

e). Find the ratio $\frac{\theta_{1,2}}{\beta}$ ($\frac{\theta_1}{\beta}$ or $\frac{\theta_2}{\beta}$) in terms of η. Here $\theta_{1,2}$ represents each of the image positions in d.) and η stands for the ratio $\frac{\beta}{\theta_E}$. (2 points) f). Find also the values of magnifications $\dfrac{\Delta\theta}{\Delta\beta}$ in terms of η for $\theta = \theta_{1,2}$ ($\theta = \theta_1$ or $\theta = \theta_2$), when $\Delta\beta \ll \beta$, and $\Delta\theta \ll \theta$. (1 point)